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On Stochastic Einstein Locality in Algebraic Relativistic Quantum Field Theory

F. A. Muller[1]

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This paper assesses Miklós Rédei's [1991] proof of the proposition that algebraic relativistic quantum field theory is stochastic Einstein local. The conclusion is that either Rédei's proof is spurious, in that it does not really prove what it intends to establish, or that the proof is fallacious. The paper is self-contained in the sense that the few ingredients of algebraic quantum theory that go into Rédei's proof are first summed up. Then Hellman's definition of stochastic Einstein locality is discussed, a detailed exposition is offered of Rédei's proof, and finally the author's refutation is explicated.

1. ALGEBRAIC RELATIVISTIC QUANTUM FIELD THEORY

These introductory remarks on algebraic relativistic quantum field theory (AQT) are based on Haag (1992) and Landsman (1991).

In AQT one starts with the uniform closure

$$\mathscr{A}_{\rm loc} := \bigcup_G \mathscr{A}(G)$$

called a *net*, where G is an open bounded subset of the Minkowski space-time manifold $\mathscr{M}$ from special relativity, and $\mathscr{A}(G)$ is a C^*-algebra of *local observables on* G. The properties of this net $\mathscr{A}_{\rm loc}$ follow from the axioms of AQT. A *state* ψ is a normalized, continuous, positive linear functional on the local algebra, mapping observables to complex numbers $[\psi: \mathscr{A}(G) \mapsto \mathbb{C}]$, in such a way that $\psi(\mathscr{A}(G))$ is the *expectation* value of observable $A(G) \in \mathscr{A}(G)$ in state ψ. Let $\mathscr{F}$ be the set of states. A Heisenberg-like picture is employed: the states are, in the terminology of AQT,

[1]Faculty of Physics and Astronomy, Department of Foundations of Science, University at Utrecht, P.O. Box 80 000, 3508 TA Utrecht, The Netherlands.

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nonlocal, or *global*, while the local observables carry the space-time dependence. The Gel'fand–Naimark–Segal construction achieves a correspondence between states and *cyclic representations* of a C^*-algebra $\mathscr{A}(G)$. We mention the fact that the Hilbert space enters at the level of representation (of the local algebra) as the so-called *representation space*. The details of the construction of the representation from the state, and of many other things, need not concern us at the present moment; what does concern us, because used by Rédei in his proof, are the following two points.

First, in orthodox von Neumann–Dirac quantum mechanics the probability of finding value a when measuring a physical magnitude represented by a (nondegenerate) operator $\hat{A}$ of a system in (normalized) state $|\psi\rangle \in \mathscr{H}$ equals the expectation value of the projector $\hat{P}_a := |a\rangle\langle a|$ in state $|\psi\rangle$, where $\hat{A}|a\rangle = a|a\rangle$:

$$\text{Prob}([\hat{A}]^{|\psi\rangle} = a) = \langle\psi|\hat{P}_a|\psi\rangle$$

In the algebraic approach the probability of finding a value a when measuring local observable $A(G) \in \mathscr{A}(G)$ in state $\psi \in \mathscr{F}$ equals the expectation value of local projection $P_a(G) \in \mathscr{A}(G)$ in state $\psi \in \mathscr{F}$:

$$\text{Prob}([A(G)]^{\psi} = a) = \psi(P_a(G)) \tag{1}$$

Both ways of doing things yield the same number; the difference is that the piece of measurement apparatus associated with some physical magnitude, performing the measurement in space-time region G, is stipulated by ordinary quantum mechanics to measure the operator $\hat{A}$ and by the algebraic approach to measure the local observable $A(G)$.

The second point concerns the implementation of the Poincaré group $\mathscr{P}$ in AQT (actually the covering group $\bar{\mathscr{P}}$ in order to deal with spin—we gloss over this point). Poincaré symmetry in AQT means that to each Poincaré transformation $g := (\Lambda, b)$, where Λ denotes a Lorentz transformation and b a space-time translation, there corresponds an automorphism on the net $\alpha_g : \mathscr{A}(G) \mapsto \mathscr{A}(gG) \in \mathscr{A}_{\text{loc}}$. The local algebra $\mathscr{A}(G)$ of region G is mapped to the local algebra $\mathscr{A}g(G)$ of the Poincaré transformed region $g(G)$ in such a way that all algebraic relations between the observables are conserved (form invariance) and that all scalar quantities, like expectation values (and thus probabilities), are invariant. The states and observables in AQT transform by $g \in \mathscr{P}$ as follows (the existence of inverse transformations is a group property):

$$\psi \mapsto \psi' = \psi \circ \alpha_g^{\text{inv}} \qquad \text{and} \qquad A \mapsto A' = \alpha_g \circ A \circ g^{\text{inv}} \tag{2}$$

It is easy to see that the expectation value of any observable $A(G) \in \mathscr{A}(G)$ in any region $G \subset \mathscr{M}$ in any state $\psi \in \mathscr{F}$ is invariant under any Poincaré transformation $g \in \mathscr{P}$:

$$\psi'(A'(G')) = (\psi \circ \alpha_g^{\text{inv}})[(\alpha_g \circ A \circ g^{\text{inv}})(g(G))] = \psi(A(G))$$

Actually the transformations (2) of AQT follow from the Poincaré invariance of scalar quantities applied to the expression for the expectation values as we just did. Applying (2) to $P_a(G) \in \mathscr{A}(G)$ yields for $\forall \psi \in \mathscr{F}$

$$\psi(\alpha_g[P_a(G)]) = \psi(P'_a(g(G))) \tag{3}$$

One might wonder: where are the quantum fields? The transition from an algebra of local observables to a realistic field theory is notoriously difficult and one of the main areas of interest of AQT; connection between the two is established by (internal, dynamical) superselection rules and the concomitant symmetries. We shall not be needing the fields in this paper. To focus the mind through AQT glasses: think about any microphysical system at G, of which any physical magnitude is represented by an element $A(G)$ of the local algebra $\mathscr{A}(G)$, which can be measured by an A-apparatus at G.

2. STOCHASTIC EINSTEIN LOCALITY

Hellmann (1982*a*) aims to close the gap between the physical requirements of special relativity, especially the no-superluminal-action requirement, and the idea of "Bell-locality," which is the notion that (the probability of) measurement outcomes (are functions which) do not depend on whatever there is outside the light-cone of the measurement event. A theory is *deterministic in magnitude* $A(z_1, \ldots, z_n)$, $z_i \in \mathscr{M}$, iff any two models of the theory that agree evaluated at all k-tuples $y_1, \ldots, y_k$ ($y_i \in \mathscr{M}$) for all $k \in \mathbb{N}$, earlier[2] than an arbitrary n-tuple $x_1, \ldots, x_n$ ($x_j \in \mathscr{M}$), also agree with regard to $A(x_1, \ldots, x_n)$. A theory is *deterministic* iff it is deterministic in all its magnitudes. Call the intersection of an infinite space-like hypersurface dividing $\mathscr{M}$ into two disjoint parts and the backward light-cone of any subset $V \subset \mathscr{M}$ (which may contain only one point) a *spacelike backward light-cone slice of* V, or for short a V*-splice*. A theory is *deterministic Einstein local* (DEL) *in magnitude* $A(z_1, \ldots, z_n)$ iff any two models of the theory that agree evaluated at all k-tuples $y_1, \ldots, y_k$ ($y_i \in \mathscr{M}$), for all $k \in \mathbb{N}$, on an arbitrary $x_1, \ldots, x_n$-splice, agree with regard to $A(x_1, \ldots, x_n)$.[3] Hellman (1982*a*, p. 455)) shows that every deterministic theory that obeys

[2]Hellman (1982*a*, p. 458) takes this definition relative to an inertial frame.

[3]Backward light-cone $LC^-(x_1, \ldots, x_n)$ is then the union of the backward light-cones: $\bigcup_i LC^-(x_i)$.

DEL implies a Bell inequality. And Hellman (1982*a*, p. 450) claims "that any genuine physical theory, deterministic in its physical magnitudes, will have to satisfy this model-theoretic condition [DEL] if it is to avoid commitment to action-at-a-distance". Indeed, if two models match in a backward light-cone but differ at the top of this light-cone, then the determining factor for this difference (the theory is deterministic) must lie outside the light-cone and hence propagates superluminally.

Hellman (1982*b*) aims to formulate a condition analogous to DEL for stochastic theories, called *stochastic Einstein locality* (SEL), and purports to show that SEL does not imply a Bell inequality and that quantum mechanics does not violate SEL. We shall not be concerned with the latter two points, only with the first one. Here is an:

Informal Definition. A theory is *stochastic Einstein local* (SEL) in magnitude $A(z_1, \ldots, z_n)$, where z_i are space-time points, iff any two models of the theory which agree evaluated on an arbitrary $x_1, \ldots, x_n$-splice, for an arbitrary k-tuple, also agree with regard to the probability of finding value a for magnitude $A(x_1, \ldots, x_n)$, for all possible values of $A(x_1, \ldots, x_n)$, but exclude from this probability all conditional probabilities of finding value a for $A(x_1, \ldots, x_n)$ that (i) can be derived from any joint probability which is *locally determined* by the theory (meaning: any two models of the theory which agree evaluated on a splice of the intersection of the backward light-cones of the events of this joint probability under consideration also agree with regard to this joint probability); and that (ii) that can be derived by using information outside the backward light-cone of $x_1, \ldots, x_n$ in order to assign a new state to the system under consideration at $x_1, \ldots, x_n$.

The provisos (i) and (ii) do not come into play in Rédei's proof, so a formal account of them will not be given—in contradistinction to the central part of SEL. But their idea is this (see Fig. 1). Hellman (1982*b*, p. 467) deems proviso (i) necessary to avoid that "virtually no theory with essentially stochastic elements could satisfy locality [SEL]." An example that would turn orthodox quantum mechanics into an SEL-violating theory if proviso (i) were not included is Einstein's remark that if *one* particle leaves a decaying nucleus and is detected somewhere, then the probability of finding it somewhere else drops instantaneously to zero. But, as Hellman continues, "there is no basis for inferring that some energy or force has propagated faster than light." Hellman (1982*b*, p. 478) motivates proviso (ii) by the idea that in quantum mechanics it is "an unresolved issue how to understand reduction" of the state to an eigenstate (the projection postulate). One could add that there is no such thing as a physical interaction, as far as detectors can tell, between the spacelike

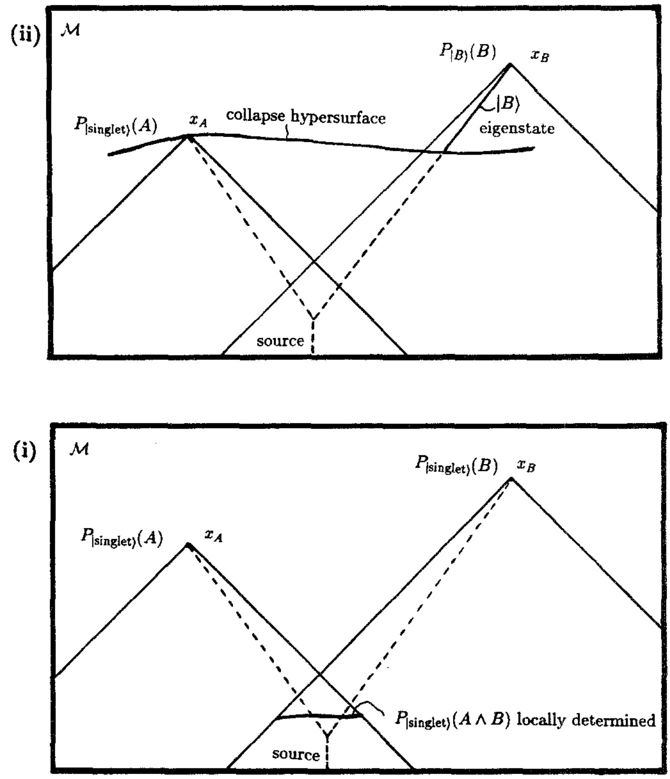

Fig. 1. Hellman's provisos of stochastic Einstein locality in the Bell experiment.

separated particles in the Bell experiment when a measurement is performed on either one particle. Thus everything in orthodox quantum mechanics that *prima facie* may turn quantum mechanics into a nonlocal theory, in the sense of violating SEL, is explicitly excluded by means of these provisos. Small wonder that Hellman (1982*b*, p. 479) can prove that quantum mechanics *without the projection postulate* is a stochastic Einstein local theory. Tailor-made suits always fit.

Hellman takes a theory **T** to be a consistent set of sentences (propositions) formulated in a formal language $\mathscr{L}(\mathbf{T})$. In the formal definition of SEL we shall encounter quantification over relations and functions (both are logically speaking predicates), so the syntaxis of $\mathscr{L}(\mathbf{T})$ must be at least as rich as second-order predicate logic. For the semantics of $\mathscr{L}(\mathbf{T})$ we shall choose from the set of all admissible bivalent valuations, which are mappings $\mathscr{L}(\mathbf{T}) \mapsto \{\mathsf{T}, \mathsf{F}\}$. As usual $\mathbf{T} \vDash S$ means: all admissible valuations that satisfy **T** (render **T** True), satisfy sentence S too. A theory $\mathbf{T} \subset \mathscr{L}(\mathbf{T})$ is (by definition) closed under derivation (if $\mathbf{T} \vdash S$, then $S \in \mathbf{T}$), where the notation of derivability ($\vdash$) is defined, for example, by Gentzen's system of Natural Deduction (Van Dalen, 1983, p. 156). Mod(**T**) abbreviates the proper class of all models of **T** (it is not a set, so we shall use ϵ as an abbreviation of 'belongs to'). Sent($\mathscr{L}(\mathbf{T})$) is the set of sentences of $\mathscr{L}(\mathbf{T})$. An explication of what is meant here by 'a model of theory **T**' will not be given; we only need to know the soundness property: all derivable sentences from the theory are satisfied (rendered true) by all models:

$$\forall M \epsilon\, \mathrm{Mod}(\mathbf{T}), \quad S \in \mathrm{Sent}(\mathscr{L}(\mathbf{T})): \quad \mathbf{T} \vdash S \Rightarrow M \vDash S,$$
$$\text{or equivalently,} \quad M \vDash \mathbf{T} \tag{4}$$

Below $LC^-(G)$ is the backward light-cone of $G \subset \mathscr{M}$; $[A(x_1, \ldots, x_n)]^\phi \in a$ means: the measured value of $A(x_1, \ldots, x_n)$ in state ϕ lies in a; Sp(A) is the set of allowed values of magnitude A by the theory; we employ Putnam's 'bastard notation': the variables of the logical universe of the model are replaced by their interpretations (the space-time points of the Minkowski manifold $\mathscr{M}$); and employ another bastard notation: for instance, $f(x) = r$ refers, besides to the obvious, also to the statement "the function f maps x onto r," so we use the same symbols for the elements of mathematical reality as for their names in $\mathscr{L}(\mathbf{T})$. $\mathrm{Pred} := \bigcup_n \mathrm{Pred}(n) \subset \mathrm{Form}(\mathscr{L}(\mathbf{T}))$ ($n \in \mathbb{N}$), where Pred(n) is the set of n-ary predicates, naming the n-ary relations R^n and the $(n-1)$-ary functions f^{n-1}, and Form($\mathscr{L}(\mathbf{T})$) is the set of well-formed linguistic expressions (formulas) of the language $\mathscr{L}(\mathbf{T})$. The physical magnitude $A(x_1, \ldots, x_n)$ is treated as an $(n+1)$-ary predicate. Now we are ready for the:

Formal Definition. A theory **T** is *stochastic Einstein local* (SEL) in magnitude

$A(z_1, \ldots, z_n)$, $z_i \in \mathscr{M}$, iff

$\forall M, M' \epsilon\, \mathrm{Mod}(\mathbf{T})$, $\forall x_1, \ldots, x_n$-splice $\subset LC^-(x_1, \ldots, x_n) \subset \mathscr{M}$,

$\forall k \in \mathbb{N}$, $\forall k$-tuples $y_1, \ldots, y_k$ on this splice:

if $\forall R^k \in \mathrm{Pred}(k)$, $\forall f^k \in \mathrm{Pred}(k+1)$, $\forall r \in \mathbb{R}$:

$$M \vDash R^k(y_1, \ldots, y_k) \Leftrightarrow M' \vDash R^k(y_1, \ldots, y_k) \text{ and}$$
$$M \vDash f^k(y_1, \ldots, y_k) = r \Leftrightarrow M' \vDash f^k(y_1, \ldots, y_k) = r,$$

then $\forall a \subset \mathrm{Sp}(A) \subset \mathbb{R}$, $\forall$state ϕ:

$$M \vDash \mathrm{Prob}([A(x_1, \ldots, x_n)]^\phi \in a) = p \Leftrightarrow$$
$$M' \vDash \mathrm{Prob}([A(x_1, \ldots, x_n)]^\phi \in a) = p,$$

under provisos (i) and (ii).

Remarks. The application of SEL to quantum mechanics is not so straightforward as Hellman claims—who takes for the magnitude A in the Bell experiment the observable spin of particle 1 along **a** $[\hat{\sigma}_{\mathbf{a}}(1)]$ and for $z_1 = z_A$ $(n = 1)$, where $z_A \in \mathcal{M}$ is the place-time of the spin measurement event—, because observables in quantum mechanics are not functions on space-time points, but operators on Hilbert space. So $A(z_A)$ should be understood in this case as: a measurement event of observable $\hat{A}$ at $z_A \in \mathcal{M}$, predicating space-time point z_A with value $\pm\frac{1}{2}\hbar$—instead of the particle!

Another point is that Hellman (1982*a*, p. 448) assumes that "**T** specifies a background ontology of Minkowski space-time: every model M of **T** contains a manifold $\mathcal{M}$ of 'events'." This is obviously not the case in quantum mechanics, though the above definition of SEL is straightforwardly applied to quantum mechanics by Hellman (1982*b*, part 4). If one wants to add a space-time arena to quantum mechanics, for instance, to locate measurement-events, this arena has to be the space-time manifold of Newtonian physics, because the space-time symmetry group of quantum mechanics is not the Lorentz group, but the Galilei group. It is, however, not easy to understand why we should fear superluminal action in a space-time arena which lacks a light-cone structure.

3. RÉDEI'S PROOF

Miklós Rédei (1991) offers a proof that AQT obeys the SEL criterion. To prove this, Rédei first defines what it means for AQT to be a SEL theory. Below $\mathrm{Sp}(A(G))$ is the spectrum of observable $A(G)$ and $G_1, \ldots, G_k$ a k-tuple of open, bounded space-time regions in $\mathcal{M}$.

Formal Definition. AQT is *stochastic Einstein local* (SEL) in $G \subset \mathcal{M}$ iff

$\forall A(G) \in \mathscr{A}(G)$, $\forall M, M' \epsilon$ Mod(AQT),

$\forall k \in \mathbb{N}$, $\forall k$-tuples $G_1, \ldots, G_k \subset LC^{-}(G) \subset \mathcal{M}$:

if $\forall R^k \in \mathrm{Pred}(k)$, $\forall f^k \in \mathrm{Pred}(k+1)$, $\forall r \in \mathbb{R}$:

$$M \models R^k(G_1, \ldots, G_k) \Leftrightarrow M' \models R^k(G_1, \ldots, G_k) \text{ and}$$

$$M \models f^k(G_1, \ldots, G_k) = r \Leftrightarrow M' \models f^k(G_1, \ldots, G_k) = r,$$

then $\forall a \subset \mathrm{Sp}(A(G)) \subset \mathbb{R}$, $\forall \psi \in \mathscr{F}$:

$$M \models \mathrm{Prob}([A(G)]^{\psi} \in a) = p \Leftrightarrow M' \models \mathrm{Prob}([A(G)]^{\psi} \in a) = p$$

under provisos (i) and (ii).

Again, AQT is SEL iff it is SEL in all $G \subset \mathscr{M}$. This definition of SEL in the context of AQT is not identical to Rédei's definition; we have added a few things for clarity. (a) Rédei omits the state $\psi \in \mathscr{F}$. But it must be added, because the probability of an observable having a certain value depends on the state. (b) Rédei takes the probabilities of finding the value of a local observable $A(G)$ *in an interval* $a \subset \mathrm{Sp}(A(G))$, for all subsets of the spectrum of the observable, whereas Hellman takes probabilities of finding *exactly one value* a of an observable, for all values in the spectrum of the observable (as we have done). So, formally Rédei's definition accommodates observables with continuous spectra too. (c) The provisos are not mentioned by Rédei; they should be, in order to prevent spurious violations of SEL.

Next we arrive at the central result:

Proposition. *Algebraic Relativistic Quantum Field Theory is a stochastic Einstein local theory.*

Rédei's Proof. The proof is a *reductio ad absurdum* argument. First a rough sketch, then the rigorous proof.

Assume AQT and not SEL, that is, assume the antecedent of SEL and the negation of the consequent. Now push the region G into its backward light-cone by an active space-time translation $b \in \mathscr{P}$ [$bG \subset LC^{-}(G)$] and consider the local algebra on the shifted region [$\mathscr{A}(bG)$] which is related to the local algebra on the original region G by an automorphism (α_b) on the net ($\mathscr{A}_{\mathrm{loc}}$). Poincaré symmetry requires that the probabilities on G concerning any observable of the local algebra [$A(G) \in \mathscr{A}(G)$] are equal to the probabilities concerning the corresponding observable of the local algebra on the shifted region [$A'(bG) = \alpha_b[A(G)] \in \mathscr{A}(bG)$]. So if a certain probability is different in two models, as follows from the negation of the consequent of SEL, it remains different after applying a Poincaré transformation to the situation. By mathematical manipulation this probability is construed as a real scalar function on the shifted region ($h_b : \mathscr{M} \supset bG \mapsto [0, 1] \subset \mathbb{R}$), which consequently means that one function ascribes different values to the shifted region in the two models. But by supposition of SEL's antecedent all corresponding functions of the two models ascribe identical values to identical regions in the backward light-cone of G. Contradiction.

Now the rigor. Assume that AQT violates SEL. Then the antecedent of SEL is true by assumption. Any space-time translation is a Poincaré transformation: $b \in \mathscr{P}$. Let $b^{\mu} \in \mathbb{R}^4$ be a timelike vector pointing backward; choose $\|b^{\mu}\|$ sufficiently large such that it shifts G into its own backward light-cone: $bG \subset LC^{-}(G)$. The second conjunct of SEL's antecedent certainly holds for the special case $G_1 = bG$ $(k = 1)$. $\forall M, M' \epsilon$ Mod(AQT), $\forall f \in$Pred(2):

$$M \models f(bG) = r \Leftrightarrow M' \models f(bG) = r \tag{5}$$

Now we turn to SEL's consequent. Either $\exists a \in \mathrm{Sp}(A(G))$, $\exists \psi \in \mathscr{F}$, such that the following is false:

$$M \models \mathrm{Prob}([A(G)]^{\psi} \in a) = p \Leftrightarrow M' \models \mathrm{Prob}([A(G)]^{\psi} \in a) = p$$

or one of the provisos is false. Since the provisos are completely ignored by Rédei, we shall interpret him as assuming, for *reductio*, that a genuine violation of SEL occurs. In other words, we interpret Rédei charitably, so that only spurious violations are not accounted for by his proof. The negation of SELs consequent is then true, which yields

$$M \models \mathrm{Prob}([A(G)]^{\psi} \in a) = p \wedge M' \models \mathrm{Prob}([A(G)]^{\psi} \in a) \neq p \tag{6}$$

Using (1), we obtain for this *reductio* assumption (6)

$$M \models \psi(P_a(G)) = p \wedge M' \models \psi(P_a(G)) \neq p \tag{7}$$

The state $\psi \in \mathscr{F}$ is a linear functional which maps observable $P_a(G)$ to a number in the interval $[0, 1] \subset \mathbb{R}$.

Since the space-time symmetries $\mathscr{P}$ of AQT form a group, α_b has an inverse automorphism α_b^{inv}; we obtain from (3) with $g = b$ and applying the functional ψ

$$\psi(P_a(G)) = \psi(\alpha_b^{\mathrm{inv}}[P'_a(bG)]) \tag{8}$$

The right-hand side of (8) defines a mapping h_b from $bG \subset \mathscr{M}$ to $[0, 1] \subset \mathbb{R}$ [the left-hand side of (8) is a probability due to (1); so h_b is a real scalar function on a region of the manifold]:

$$h_b\colon \ \mathscr{M} \mapsto \mathbb{R}, \quad h_b(bG) = \psi(P_a(G)) \qquad \text{where} \quad h_b := \psi \circ \alpha_b^{\mathrm{inv}} \circ P'_a \tag{9}$$

Substituting in (7) yields

$$M \models h_b(bG) = p \wedge M' \models h_b(bG) \neq p \tag{10}$$

Since (5) holds $\forall f \in$Pred(2), we are allowed to choose the special case $f = h_b$:

$$M \models h_b(bG) = r \Leftrightarrow M' \models h_b(bG) = r \tag{11}$$

Whether $p = r$ or $p \neq r$, on both accounts (10) contradicts (11). To avoid this absurdity, it must be assumed that AQT does not violate SEL. QED

Rédei's proof is far more compact, which made it not that easy to see where it goes wrong; the more detailed exposition above will enable us to point out where and to understand why the proof is either spurious or fallacious.

4. REFUTATION OF RÉDEI'S PROOF

To start with, we remark that the proof effectively uses only the symmetry of the group of space-time translations—which constitutes together with the Lorentz group the Poincaré group $\mathscr{P}$—so the proof holds, too, for an algebraic field theory having the Galilei group as its space-time symmetry group. But surely a nonrelativistic field theory need not obey SEL.

Furthermore, Poincaré symmetry by itself does not exclude tachyon fields; they can be introduced without destroying the Poincaré symmetry. With tachyon fields it is easy to think of a model that violates SEL by triggering a tachyon source in a region $G_{\text{tach}} \subset \mathscr{M}$ at spacelike distance from the region $G \subset \mathscr{M}$; then the probabilities at G will definitely be influenced by G_{tach}; hence a violation of SEL. Tachyon fields are excluded, however, by an axiom of AQT stating that all energies are positive (Haag, 1992, p. 106). But this axiom is nowhere used in the proof.

Another crucial ingredient one expects to appear in the proof is AQTs axiom of local commutativity, which states that the algebras of spacelike separated regions commute (Haag, 1992, p. 107). But it is absent from the proof too.

With the above-mentioned axioms of AQT it may well be possible to prove that AQT obeys SEL, but the considerations above already strongly indicate that this cannot be established by Rédei's proof. First we shall detect a logical oversight in the proof, which renders the proof vacuous.

The assumption that SELs consequent is false, in that the probability of an observable $A(G)$ having value a in state ψ is different in two models, generates no problem. But setting these probabilities equal to the expectation value of the local observable $P_a(G) \in \mathscr{A}(G)$, as stipulated by AQT (1), generates a major problem, because AQT *defines* the state $\psi \in \mathscr{F}$ as the functional which maps $P_a(G)$ to its one and only expectation value. Assume it equals $p \in [0, 1]$:

$$\text{“}\psi(P_a(G)) = p\text{”} \in \text{AQT}$$

Then due to (7) clearly M' is *not* a model of AQT, for M' renders this statement false. If we assume that this expectation value does not equal p, but $p'(\neq p)$, such that M' does not turn out to be a model of AQT, then M is not a model of AQT, again due to (7). So statement (7) is inconsistent with the assumption that both M and M' are models of AQT. Having hardly asserted the *reductio* assumption (7), one can immediately conclude that it contradicts AQT, given the soundness property (4) of the models. So the detour with the space-time translation $b \in \mathscr{P}$, *which is the actual content of the proof*, is superfluous.

Furthermore, even SELs antecedent has not been used! So this 'proof' goes through if we replace SELs antecedent by: any two models that *disagree* in all their functions and relations in the backward light-cone of $G \subset \mathscr{M}$. Then Rédei's proof justifies the following proposition, too: if two models of AQT that *disagree* in *all* their functions and relations in the backward light-cone of $G \subset \mathscr{M}$, then they *agree* with regard to all the expectation values of all local observables of $\mathscr{A}(G)$ at G. Quite a miracle!

The conclusion is that this is an empty proof. The only aspect of AQT it effectively uses is that AQT defines the state by means of an expectation value. Every aspect of AQT that smells of locality (the Lorentz group, the so-called diamond property, local commutativity, which are explicitly added to AQT to meet the exigencies of special relativity) is ignored in Rédei's proof.

Is there no way of retaining both M and M' as models of AQT? Yes there is.[4] If the expectation values of $P_a(G)$ in the two models M and M' are different, then according to AQT they just belong to different states, say ψ and ϕ, respectively. On this account both M and M' still are models of AQT and (7) is saved. The Rédei procedure brings us in this case to *two different* scalar functions on bG:

$$M \vDash h_b(bG) = \psi(P_a(G)), \qquad h_b := \psi \circ \alpha_b^{\text{inv}} \circ P'_\alpha$$

$$M' \vDash q_b(bG) = \phi(P_a(G)), \qquad q_b := \phi \circ \alpha_b^{\text{inv}} \circ P'_a$$

If some system under consideration is at bG in state ψ according to model M and in state ϕ according to model M', then we do arrive at statement (10). This is presumably what Rédei had in mind. But we cannot arrive logically foolproof at a contradiction between statements (10) and (5), as needed for the *reductio* argument. To see this, note the following.

In general, the values of Poincaré transformed functions on Poincaré transformed regions of the Poincaré transformed situation are equal to the values of the original functions on the original regions in the original situation: $f(bG) = f_b(b(bG))$ and $h(G) = h_b(bG)$, where $f_b = f \circ b^{\text{inv}}$ and $h_b = h \circ b^{\text{inv}}$. But Poincaré transformed functions on Poincaré transformed regions of the Poincaré transformed situation are not supposed to be equal to the original functions on the Poincaré transformed regions of the original situation: $f_b(b(bG)) \neq f(b(bG))$ and $h_b(bG) \neq h(bG)$. To suppose this, would be a mistake. It is exactly the mistake Rédei makes by

[4]Dennis Dieks pointed to this way out of the preceding criticism.

supposing that the b-transformed function h_b on the b-transformed region bG is equal to an original function f on the b-transformed region $bG \subset \mathcal{M}$ of the original situation: $h_b(bG) \neq f(bG)$. The illegitimate choice $f = h_b$ just before statement (11) is the culprit, as well as the tacit identification $b(G) = bG$. Therefore (5) does not entail (11) and the desired contradiction between statements (5) and (10) simply does not follow. To conclude, by choosing different states in order to avoid a spurious proof, the reasoning turns out to be fallacious.

We close with the remark that in a more encompassing treatment of SEL in the context of AQT, which contains a correct proof of the central Proposition of the present paper, the latter fallacy of Rédei's proof will be treated in formal detail. See Muller and Butterfield (1994).

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